Journal of Financial Economics 128 (2018) 1–15

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Absolving beta of volatility's effects[☆]

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ARTICLE INFO

Article history:
Received 13 January 2017
Accepted 11 February 2017
Available online 2 February 2018

JEL classification:
G12
G14

Keywords:
Beta
Anomaly
Volatility

ABSTRACT

The beta anomaly, negative (positive) alpha on stocks with high (low) beta, arises from beta's positive correlation with idiosyncratic volatility (IVOL). The relation between IVOL and alpha is positive among underpriced stocks but negative and stronger among overpriced stocks (Stambaugh, Yu, and Yuan, 2015). That stronger negative relation combines with the positive IVOL-beta correlation to produce the beta anomaly. The anomaly is significant only within overpriced stocks and only in periods when the beta-IVOL correlation and the likelihood of overpricing are simultaneously high. Either controlling for IVOL or simply excluding overpriced stocks with high IVOL renders the beta anomaly insignificant.

1. Introduction

The beta anomaly is perhaps the longest-standing empirical challenge to the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) and asset-pricing models that followed. Beginning with the studies of Black et al. (1972) and Fama and MacBeth (1973), the evidence shows that high-beta stocks earn too little compared to low-beta stocks. In other words, stocks with high (low) betas have negative (positive) alphas. Explanations of the beta anomaly typically identify beta as the relevant stock characteristic generating the anomaly.

We find that beta is not the stock characteristic driving the beta anomaly. Rather, beta suffers from guilt by association. Specifically, in the cross-section of stocks, the correlation between beta and idiosyncratic volatility (IVOL) is positive, about 0.33 on average. This correlation can exist for a number of reasons. Greater leverage can increase both IVOL and beta on a company's stock. Also, if high-IVOL stocks are more susceptible to mispricing, part of which arises from market-correlated sentiment, then that source of market sensitivity is greater for high-IVOL stocks. The beta-IVOL correlation produces the beta anomaly because IVOL is related to alpha. The alpha-IVOL relation involves mispricing, as shown by Stambaugh et al. (2015). The relation between alpha and IVOL is positive among underpriced stocks but negative and stronger among overpriced stocks, where a stock's mispricing is measured by combining its rankings with respect to 11 prominent return anomalies. As that study explains, the dependence of the direction of the alpha-IVOL relation on the direc-

☆ We are grateful for comments from Li An, Wesley Gray, Michael O'Doherty, Nikolai Roussanov, Jay Shanken, and participants in the 2017 Mid-Atlantic Research Conference in Finance, the 2017 China International Conference in Finance, the 2017 Western Finance Association Meeting, and seminars at the Shanghai Advanced Institute of Finance, Tulane University, the University of Chicago, and the University of Pennsylvania. Yuan gratefully acknowledges financial support from the NSF of China (71522012).

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https://doi.org/10.1016/j.jfineco.2018.01.003
0304-405X/

tion of mispricing is consistent with IVOL reflecting arbitrage risk that deters price correction. The stronger negative relation among overpriced stocks is consistent with less capital available to bear the arbitrage risk of shorting overpriced stocks as compared to the capital that can bear such risk when buying underpriced stocks. The asymmetry in the strength of the positive and negative relations produces a negative alpha-IVOL relation in the total stock universe. That negative relation combines with the positive correlation between beta and IVOL to produce the negative relation between alpha and beta, the beta anomaly.

Consistent with our explanation, we find a significant beta anomaly only within the most-overpriced stocks, i.e., those in the top quintile of the Stambaugh et al. (2015) mispricing measure. For those stocks, the alpha spread between stocks in the top and bottom deciles of beta is −60 basis points (bps) per month, with a *t*-statistic of −2.82. Across the remaining four quintiles of the mispricing measure, the same spread ranges from −25 bps to 18 bps, with *t*-statistics between −1.28 and 0.90. These results are as expected: If the beta anomaly is due to beta's correlation with IVOL, then a negative alpha-beta relation can arise only where there is a negative alpha-IVOL relation, i.e., only among overpriced stocks. The negative alpha-beta relation for those stocks is strong enough to deliver the well-known beta anomaly when sorting on beta in the total universe. Even though the alpha-IVOL relation for underpriced stocks is significantly positive, a weaker (insignificant) corresponding positive alpha-beta relation among those stocks is unsurprising. That segment's positive alpha-IVOL relation is weaker than the negative relation among overpriced stocks, so IVOL's role in that relation survives only weakly when played imperfectly by beta.

Also consistent with our explanation, the beta anomaly becomes insignificant after controlling for IVOL. We control for IVOL in a variety of ways, including independent double-sorting on beta and IVOL as well as sorting on the component of beta that is cross-sectionally orthogonal to IVOL. Deleting high-IVOL overpriced stocks, just 7% of the stock universe (1% in terms of market value), also renders the beta anomaly insignificant. In contrast, deleting the 7% of stocks with the highest betas (5% in terms of market value) has virtually no effect on the beta anomaly.

Beta-driven explanations of the beta anomaly seem challenged by our finding that the anomaly exists only among the most-overpriced stocks. For example, the most familiar beta-driven explanation of the beta anomaly argues that borrowing and/or margin constraints confer an advantage to high-beta stocks for which investors accept lower returns (e.g., Black, 1972; Fama, 1976; Frazzini and Pedersen, 2014). If some investors prefer high-beta stocks for that reason, it is not clear why such investors should prefer only the high-beta stocks that are overpriced for reasons unrelated to beta.[1] One might think such investors would instead, ceteris paribus, prefer the underpriced high-beta stocks. Other beta-driven explanations that face the same challenge include preferences for high-beta stocks by unsophisticated optimistic investors (e.g., Barber and Odean, 2000; Antiniou et al., 2016) and by institutional investors striving to beat benchmarks (e.g., Baker et al., 2011; Christoffersen and Simutin, 2017). Similarly, the same challenge confronts the explanation proposed by Hong and Sraer (2016). They suggest the beta anomaly stems from short-sale impediments combined with the greater sensitivity of high-beta stocks to disagreement about the stock market's prospects, but again it is not clear why such an effect should be confined to overpriced stocks.

Although alternative explanations of the beta anomaly seem inconsistent with our evidence, the underlying mechanisms accompanying such explanations may still be at work in making characteristics other than beta relevant to the cross-section of returns. For example, while borrowing constraints appear not to be the explanation for the beta anomaly, those constraints may nevertheless exert return effects when investigated using a measure other than beta. In fact, Asness et al. (2016) conclude that a stock's correlation between its return and the market return is related to average return in ways consistent with borrowing constraints.

We also examine IVOL's role in the betting-against-beta (BAB) strategy of Frazzini and Pedersen (2014). The BAB strategy buys low-beta stocks and shorts high-beta stocks, consistent with exploiting the beta anomaly. At the same time, however, the strategy takes a levered net-long position to achieve a zero beta, thereby creating a component of the BAB strategy unrelated to the beta anomaly. As a result, the BAB strategy can produce positive alpha where there is no beta anomaly but zero alpha where there is. In fact, we find significant BAB alphas in the four mispricing quintiles that exhibit little or no beta anomaly, but we find no significant BAB alpha in the quintile that by far exhibits the strongest beta anomaly: the quintile containing the most-overpriced stocks. The BAB strategy's unlevered component, which goes long and short equal amounts of low- and high-beta stocks, isolates the contribution of the beta anomaly. The alpha on this unlevered component does not survive a control for IVOL in which we augment the three Fama and French (1993) factors with the return on a betting-against-IVOL strategy constructed analogously to the BAB strategy's unlevered component.

Our explanation of the beta anomaly requires a substantial presence of overpriced stocks along with a positive correlation between beta and IVOL. Without overpricing, there is no role for IVOL in deterring the correction of overpricing, so there is no negative alpha-IVOL relation. That negative relation does not produce the beta anomaly without a positive beta-IVOL correlation, especially within the overpriced stocks. In other words, the conditions most conducive to the beta anomaly are a substantial presence of overpriced stocks coupled with a high beta-IVOL correlation within those stocks. We pursue further support of our explanation of the beta anomaly by exploiting variation over time in both the likelihood of overpriced stocks, proxied by the Baker and Wurgler (2006) investor sentiment index, as well as the beta-IVOL correlation. Consistent with our explanation, we find a significant beta

[1] The identification of overpriced stocks is essentially unrelated to beta. The mispricing measure typically has just a 0.07 (and statistically insignificant) cross-sectional correlation with beta, which is not one of the anomaly variables used to construct the mispricing measure.

anomaly in periods when investor sentiment and the beta-IVOL correlation are both above their median values, but we find no beta anomaly when either or both quantities are below their medians.

The rest of paper proceeds as follows. Section 2 describes our measures of mispricing, IVOL, and beta. Section 3 presents our main empirical results. Section 4 analyzes the betting-against-beta strategy. Section 5 concludes.

2. Empirical measures: mispricing, IVOL, and beta

Our study's main empirical results, presented in the next section, rely primarily on sorting stocks according to one or more measures: mispricing, IVOL, and beta. In this section we explain how we estimate each of these measures.

Our measure of mispricing follows Stambaugh et al. (2015), who construct a stock's mispricing measure each month as the average of the stock's rankings with respect to 11 variables associated with prominent return anomalies. For each anomaly variable, we assign a ranking percentile to each stock reflecting the cross-sectional sort on that variable. High ranks correspond to low estimated alpha. A stock's mispricing measure in a given month is the simple average of its ranking percentiles across the anomalies. The higher is this average ranking, the more overpriced is the stock relative to others in the cross-section. Stambaugh et al. (2015) suggest their mispricing measure be interpreted as proxying for a stock's ex ante potential to be mispriced, as opposed to capturing the mispricing that survives arbitrage-driven price correction. The latter mispricing would be reflected in estimated alpha. Those authors find that among stocks identified as overpriced (underpriced) by this mispricing measure, alpha is decreasing (increasing) in IVOL, consistent with IVOL deterring price-correcting arbitrage.

The sample for our study, obtained from the Center for Research in Security Prices (CRSP), includes all NYSE/Amex/Nasdaq common stocks having prices of at least five dollars (thus excluding typically illiquid penny stocks). We follow Stambaugh et al. (2015) in eliminating stocks for which at least five (of the 11) anomaly variables cannot be computed. As those authors report, this five-anomaly requirement eliminates about 10% of the remaining stocks. Our sample period is from January 1963 through December 2013.

We compute IVOL, following Ang et al. (2006), as the standard deviation of the most recent month's daily benchmark-adjusted returns. The latter are computed as the residuals in a regression of each stock's daily return on daily realizations of the three factors defined by Fama and French (1993): market (MKT), small-minus-big (SMB), and high-minus-low (HML). This IVOL estimate is also used by Stambaugh et al. (2015). Computing IVOL using residuals in a regression on just MKT, however, produces extremely similar results for our study, which is not surprising given that three-factor and one-factor IVOL have an average (rank or simple) correlation of 0.99.

We estimate a stock's beta by regressing the stock's monthly excess return on monthly market excess returns, with excess returns computed by subtracting the one-month US Treasury bill rate. The regression includes the lagged market return to accommodate non-synchronous trading effects:

$$r_{i,t} = a_i + \beta_{i,0} r_{m,t} + \beta_{i,1} r_{m,t-1} + \epsilon_{i,t}. \tag{1}$$

We run the regression each month over a moving window covering the most recent 60 months, requiring at least 36 months of non-missing data for the stock to be assigned a beta value for the given month. The stock's time-series beta estimate is computed as

$$\hat{\beta}_i^{ts} = \hat{\beta}_{i,0} + \hat{\beta}_{i,1}, \tag{2}$$

applying the summed-slopes procedure of Dimson (1979). To increase precision, we then follow Vasicek (1973) and shrink this time-series estimate toward one to form our beta estimate,

$$\hat{\beta}_i = \omega_i \hat{\beta}_i^{ts} + (1 - \omega_i) \times 1, \tag{3}$$

where

$$\omega_i = \frac{1/\hat{\sigma}^2(\hat{\beta}_i^{ts})}{1/\hat{\sigma}^2(\hat{\beta}_i^{ts}) + 1/\hat{\sigma}^2(\beta)}, \tag{4}$$

$\hat{\sigma}(\hat{\beta}_i^{ts})$ is the standard error of $\hat{\beta}_i^{ts}$, and $\hat{\sigma}^2(\beta)$ is an estimate of the cross-sectional variance of true betas. We compute the latter estimate as

$$\hat{\sigma}^2(\beta) = \hat{\sigma}^2_{cs}(\hat{\beta}_i^{ts}) - \overline{\hat{\sigma}^2(\hat{\beta}_i^{ts})}, \tag{5}$$

where $\hat{\sigma}^2_{cs}(\hat{\beta}_i^{ts})$ is the cross-sectional variance of $\hat{\beta}_i^{ts}$, and $\overline{\hat{\sigma}^2(\hat{\beta}_i^{ts})}$ is the cross-sectional mean of $\hat{\sigma}^2(\hat{\beta}_i^{ts})$.[2]

There are numerous approaches for estimating betas on individual stocks, and the literature does not really offer a consensus. Fama and French (1992) estimate individual stocks' betas in the same way shown in Eq. (2), regressing monthly return on the current and recent lag of the market return using a five-year rolling window and then summing the coefficients as in Dimson (1979). This specification is frequently used; a recent example is Antiniou et al. (2016). Other recent studies estimate betas using shorter windows and higher-frequency returns. For example, Hong and Sraer (2016) use a one-year window with daily returns, and they include five lags of the daily market return, applying the Dimson summed-coefficients method. This latter estima-

[2] Eq. (5) relies on the identity,

$$\text{var}\{\text{E}(\hat{\beta}_i|\beta_i)\} = \text{var}(\hat{\beta}_i) - \text{E}\{\text{var}(\hat{\beta}_i|\beta_i)\}.$$

Assuming $\hat{\beta}_i$ is unbiased, i.e., $\text{E}(\hat{\beta}_i|\beta_i) = \beta_i$, allows the left-hand side to be rewritten:

$$\text{var}\{\beta_i\} = \text{var}(\hat{\beta}_i) - \text{E}\{\text{var}(\hat{\beta}_i|\beta_i)\}.$$

Replacing the right-hand terms with their corresponding sample quantities gives the right-hand side (RHS) of (5).

tion approach is also applied by Cederburg and O'Doherty (2016) when forming beta-sorted portfolios, except that, following Lewellen and Nagel (2006), they constrain the coefficients on the three least recent lagged market returns to be equal. Frazzini and Pedersen (2014) separate correlation and volatilities when estimating beta. They estimate a stock's correlation with the market (ρ_{im}) using overlapping three-day returns over the past five years, whereas they estimate the standard deviations of the stock and the market (σ_i and σ_m) using daily returns over the past year. Beta is then estimated as $(\hat{\sigma}_i/\hat{\sigma}_m)\hat{\rho}_{im}$.

We compare our method for estimating beta to the four alternative methods noted above, each of which has been used in the recent literature addressing the beta anomaly.[3] There are many criteria one could use in evaluating beta estimates. Because our study ultimately compares high-beta stocks to low-beta stocks, we want a beta estimation method that reliably identifies which stocks have the highest betas and which have the lowest at a given time. We find that our method handles this task best.

To conduct our comparison, for a given beta-estimation method we compute each stock's out-of-sample "hedging error" in each month t, which is the difference between the stock's return and the stock's estimated beta times the market return, with the estimation window for beta ending in month $t-1$. We average these hedging errors across all stocks in the same beta decile as of the end of month $t-1$. We then compute the ratio of the variance across months of these averaged (i.e., portfolio-level) hedging errors to the variance of the market return. This ratio is computed for each beta decile. We form beta deciles five different ways, using each of the estimation methods, and then average the ratio of hedging-error variance to market variance across the five sets of beta deciles, obtaining a single value for a given estimation method within a given decile. The average of these values for the top and bottom beta deciles is lowest for our beta-estimation method. The detailed results are provided in the Appendix, where we explain that the beta-estimation method with the lowest hedging-error variance is the one having the lowest mean squared estimation error in beta.

3. Empirical results

This section presents our main empirical results. To avoid specifying restrictive parametric relations, we primarily examine differences in alphas on portfolios formed by sorting on one or more of the measures defined in the previous section. In Section 3.1, we sort on beta, confirming the well-known beta anomaly in the entire universe, but we sort as well on the mispricing measure, revealing the interaction between the beta anomaly and mispricing. That interaction is consistent with IVOL's role in generating the beta anomaly, as we discuss in Section 3.2. We provide direct evidence of IVOL's role in Section 3.3, which distinguishes between the effects of beta versus IVOL in producing alpha. Section 3.4 provides additional evidence of IVOL's role by exploiting variation over time in both investor sentiment and the beta-IVOL correlation.

3.1. Beta and mispricing

We sort stocks each month by their beta estimates, forming deciles. Independently, we sort stocks on the mispricing measure, forming quintiles. We then form 50 portfolios based on the intersection of these two sorts as well as ten portfolios based just on the beta sort. All of the portfolios are value-weighted. Panel A of Table 1 reports the average number of stocks in each of the 50 portfolios produced by the two-way sort. Panel B reports the post-ranking betas of these portfolios, estimated using a simple least-squares regression over the entire sample period. Although stocks are distributed reasonably evenly across the portfolios, we do see that high-beta stocks (decile 10) are more prevalent among the most-overpriced stocks as compared to the most-underpriced stocks (56 versus 39). Also, in Panel B, we see that the estimated beta for the top decile is somewhat higher for the most-overpriced stocks than for the most-underpriced (1.67 versus 1.34). Overall, though, the two-way independent sort appears to do a reasonable job of producing substantial dispersion in beta within each mispricing level. For the one-way beta sort, the difference in beta estimates between the top and bottom deciles is 0.92, and the corresponding differences within each of the mispricing quintiles are similar in magnitude.

Table 2 reports the portfolios' alphas computed with respect to the three factors of Fama and French (1993). The alphas in the bottom row, labeled "all stocks," decline nearly monotonically as beta increases. The difference in monthly alphas between the highest and lowest beta deciles equals −31 bps, with a t-statistic of −2.08. As discussed at the outset, this "beta anomaly," which exists within the overall stock universe, is both economically and statistically significant, and it has been the subject of much research over the years.

The other five rows of Table 2 reveal that this beta anomaly, the alpha difference between the highest and lowest beta deciles, exists only within the most-overpriced stocks. In that highest quintile of the mispricing measure, we see that the beta anomaly is −60 bps per month, with a t-statistic of −2.82. In contrast, the beta anomaly within the other four mispricing quintiles ranges between −25 bps and 18 bps, with t-statistics between −1.28 and 0.90. The contrast between the absence of the beta anomaly in these other four quintiles and the pronounced beta anomaly in the most-overpriced quintile is readily apparent in Fig. 1, which plots the alphas reported in Table 2.

Although we focus on three-factor alphas, consistent with the more recent anomaly literature, alphas computed with respect to a single market factor behave very similarly. In the overall universe, for example, the one-factor alpha difference between the highest and lowest beta deciles equals −38 bps with a t-statistic of −2.14, very close to the three-factor values of −31 bps with a t-statistic of −2.08.

[3] We also explored another approach, not used by existing studies to our knowledge, that applies Vasicek shrinkage as in (3) and (4) to the estimates using one-year of daily data. Betas estimated this way perform similarly to ours in identifying high- and low-beta stocks, but the alpha on the long-short spread created by ranking on such beta estimates is insignificant.

Table 1
Portfolios formed by sorting on mispricing score and beta: numbers of stocks and estimated betas.

The table reports the average number of stocks and the estimated market betas for portfolios formed by sorting independently on mispricing scores and pre-ranking betas. A stock's mispricing score, following Stambaugh et al. (2015), is its average ranking with respect to 11 prominent return anomalies. A stock's pre-ranking beta, based on a rolling five-year window, is estimated by regressing the stock's monthly return on the contemporaneous market return plus lagged monthly return, summing the slope coefficients, and then applying shrinkage. Panel A reports the average number of stocks in each portfolio, and Panel B reports the portfolio's beta estimated using monthly returns over the sample period, January 1963 through December 2013.

Mispricing quintile	Beta decile										Highest - Lowest
	Lowest	2	3	4	5	6	7	8	9	Highest	
Panel A: Average number of stocks											
Underpriced	43	58	59	57	55	52	50	47	45	39	
2	50	53	53	52	50	50	50	49	48	45	
3	56	49	49	49	49	48	48	48	48	48	
4	53	46	45	44	47	47	47	48	49	52	
Overpriced	39	35	36	39	41	45	47	49	52	56	
Panel B: Estimated beta											
Underpriced	0.62	0.74	0.75	0.93	0.97	1.01	1.04	1.14	1.25	1.34	0.72
2	0.61	0.76	0.89	0.96	1.03	1.08	1.12	1.15	1.31	1.42	0.81
3	0.55	0.83	0.90	1.01	1.08	1.16	1.19	1.27	1.33	1.51	0.96
4	0.58	0.80	0.95	1.03	1.09	1.16	1.23	1.30	1.43	1.54	0.96
Overpriced	0.61	0.83	0.92	1.05	1.19	1.32	1.34	1.38	1.48	1.67	1.06
All stocks	0.59	0.76	0.84	0.97	1.05	1.09	1.16	1.24	1.35	1.51	0.92

Table 2
Alphas on portfolios formed by sorting on mispricing score and beta.

The table reports alphas for portfolios formed by sorting independently on mispricing scores and pre-ranking betas. Alphas are computed with respect to the three factors of Fama and French (1993). A stock's mispricing score, following Stambaugh et al. (2015), is its average ranking with respect to 11 prominent return anomalies. A stock's pre-ranking beta, based on a rolling five-year window, is estimated by regressing the stock's monthly return on the contemporaneous market return plus one lagged monthly return, summing the slope coefficients, and then applying shrinkage. The sample period is from January 1963 through December 2013. All *t*-statistics (in parentheses) are based on the heteroskedasticity-consistent standard errors of White (1980).

Mispricing quintile	Beta decile										Highest - Lowest
	Lowest	2	3	4	5	6	7	8	9	Highest	
Underpriced	0.22	0.33	0.36	0.37	0.25	0.26	0.30	0.33	0.19	0.41	0.18
	(2.18)	(3.13)	(3.50)	(3.38)	(2.24)	(2.44)	(2.62)	(2.64)	(1.32)	(2.61)	(0.90)
2	0.29	0.19	0.08	0.13	−0.05	0.21	0.06	-0.04	0.01	0.03	−0.25
	(2.66)	(1.73)	(0.71)	(1.16)	(-0.50)	(1.77)	(0.46)	(−0.30)	(0.08)	(0.23)	(−1.28)
3	0.07	−0.11	0.00	−0.02	−0.23	0.07	−0.12	−0.20	0.15	0.01	−0.05
	(0.61)	(−0.79)	(−0.01)	(−0.18)	(-1.92)	(0.45)	(−0.97)	(−1.58)	(1.09)	(0.11)	(−0.28)
4	−0.14	−0.14	−0.32	−0.29	−0.38	−0.29	−0.18	−0.40	−0.40	−0.15	−0.01
	(−1.03)	(−1.11)	(−2.80)	(−2.22)	(−2.93)	(−2.17)	(−1.31)	(−2.79)	(−2.94)	(−0.97)	(−0.05)
Overpriced	−0.35	−0.37	−0.21	−0.66	−0.49	−0.70	−1.00	−0.74	−1.08	−0.96	−0.60
	(−2.56)	(−2.54)	(−1.52)	(−4.34)	(−3.07)	(−4.75)	(−5.89)	(−4.64)	(−6.67)	(−6.11)	(−2.82)
All stocks	0.11	0.13	0.13	0.03	−0.13	−0.01	−0.10	−0.10	−0.14	−0.20	−0.31
	(1.49)	(1.63)	(1.85)	(0.47)	(−1.91)	(−0.09)	(−1.46)	(−1.31)	(−1.61)	(−1.94)	(−2.08)

The one-factor alpha for the beta spread is −81 bps (*t*-statistic: −3.50) for the overpriced stocks but is between just −30 and 18 bps (*t*-statistics between −1.38 and 0.78) for the other mispricing quintiles, again behaving very similarly to the three-factor alphas.

Some explanations of the beta anomaly identify beta as the relevant stock characteristic driving the anomaly. For example, one explanation invokes the fact that high-beta stocks offer leverage-constrained investors increased exposure to the stock market that unconstrained investors can achieve simply through leverage (e.g., Frazzini and Pedersen, 2014). A beta anomaly then arises if constrained investors wanting increased market exposure bid up the prices of high-beta stocks relative to low-beta stocks. The results in Table 2 seem to challenge such explanations. If beta drives the beta anomaly, then why would it do so only among the most-overpriced stocks? For example, if some leverage-constrained investors prefer high-beta stocks and bid up their prices, why do they prefer to do so only for stocks that a wide range of other anomalies identify as being currently overpriced? If anything, one would think such investors would prefer to increase their stock-market exposure using high-beta positions in stocks that are otherwise underpriced, as opposed to overpriced.

Another explanation of the beta anomaly is that it disappears if one measures beta in a manner that captures beta's variation over time. For example, Cederburg and O'Doherty (2016) report that when alpha is estimated as the intercept in a regression that allows beta to depend on a number of conditioning variables, the resulting alphas on beta-sorted portfolios no longer exhibit the usual negative relation to beta. In forming beta-sorted portfo-

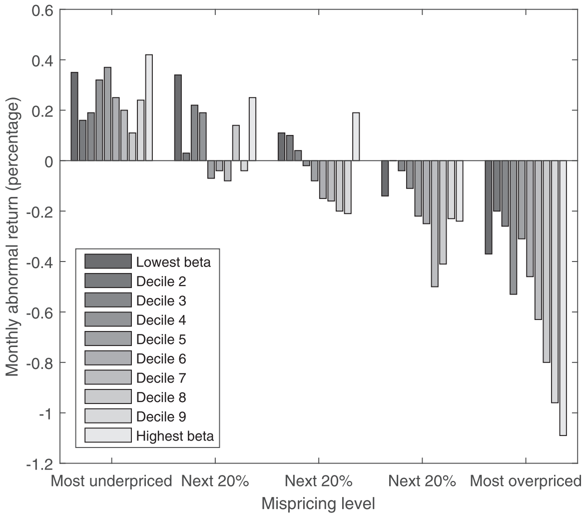

Fig. 1. Alphas for beta deciles within each mispricing quintile. The plot displays monthly alphas on value-weighted portfolios formed by the intersection of independent sorts on beta (allocated to deciles) and the mispricing measure (allocated to quintiles). Alphas are computed with respect to the three-factor model of Fama and French (1993). The sample period is from January 1963 through December 2013 (612 months).

lios, those authors estimate betas using daily returns over a one-year estimation window. As noted earlier, that beta-estimation method does not identify high- and low-beta stocks as well as the estimation method we use in forming portfolios. When we estimate alphas on our beta-sorted portfolios following the same procedure used by Cederburg and O'Doherty (2016), the results are very similar to what we report in Table 2: A significant beta anomaly exists in the overall sample as well as in the quintile of the most-overpriced stocks, but there is not a significant beta anomaly in the other mispricing quintiles. Details are provided in the Appendix.

3.2. The role of IVOL

Why is the beta anomaly confined largely to overpriced stocks? Our explanation combines two key properties of IVOL: First, beta is positively correlated with IVOL; the average cross-sectional correlation between our estimates of beta and IVOL is 0.33. Second, as shown by Stambaugh et al. (2015), IVOL has a negative relation to alpha only among overpriced stocks.

A positive correlation between beta and IVOL can exist for a number of reasons. One channel is leverage, both financial and nonfinancial. Equity returns made riskier by leverage are likely to be more sensitive to news, whether market-wide or firm-specific. For example, in the basic Black-Scholes-Merton setting analyzed by Galai and Masulis (1976), levered equity's total volatility, which includes IVOL, is proportional to the equity's beta, which increases with leverage. Another potential reason for a positive IVOL-beta correlation is behavioral. If high-IVOL stocks are more susceptible to mispricing driven by market-wide sentiment (e.g., Baker and Wurgler, 2006), and if market-wide sentiment is correlated with the market return, then returns on high-IVOL stocks have a larger market-sensitive mispricing component, increasing these stocks' betas.

The fact that IVOL has a negative relation to alpha only among overpriced stocks is consistent with IVOL reflecting risk that deters arbitrage-driven correction of mispricing. If IVOL reflects such arbitrage risk, then among underpriced stocks the alpha-IVOL relation should instead be positive, consistent with what Stambaugh et al. (2015) find. As that study explains, though, the latter positive relation is substantially weaker than the negative relation among overpriced stocks, consistent with arbitrage asymmetry. That is, many investors who would buy a stock they see as underpriced are reluctant or unable to short a stock they see as overpriced. With less arbitrage capital available to bear the risk of shorting overpriced stocks, more of the overpricing remains in equilibrium. The negative alpha-IVOL relation among overpriced stocks is thus stronger than the positive relation among underpriced stocks.

The negative alpha-IVOL relation among overpriced stocks, combined with the positive correlation between IVOL and beta, produces a negative alpha-beta relation among overpriced stocks. That relation is strong enough to produce a significant beta anomaly in the overall universe, but it is not as strong as the corresponding alpha-IVOL relation. Among the most-overpriced 20% of stocks, Stambaugh et al. (2015) report a monthly alpha difference between the highest and lowest IVOL *quintiles* equal to -150 bps with a t-statistic of -7.36, as compared to the difference in Table 2 between the highest and lowest beta *deciles* equal to -60 bps with a t-statistic of -2.82. Finding the alpha-beta relation to be weaker than the alpha-IVOL relation is as expected, given that the correlation between beta and IVOL is positive but well below one. As for the underpriced stocks, the imperfect beta-IVOL correlation is not strong enough to deliver a significant positive alpha-beta effect when combined with the relatively weaker positive alpha-IVOL relation among underpriced stocks.

Our explanation of the beta anomaly is that beta is correlated with the underlying quantity really at work: IVOL, a measure of arbitrage risk. Some studies instead argue that skewness is the underlying quantity generating both beta and IVOL anomalies. The basic explanation is that investors accept lower expected return in exchange for positive skewness while requiring higher expected return to bear negative skewness (e.g., Kraus and Litzenberger, 1976; Goulding, 2015). If the relevant measure of skewness (or co-skewness) is omitted when computing alpha but is positively correlated with beta and/or IVOL, then the latter quantities can exhibit a negative relation with alpha. Studies that empirically explore skewness as a source of the beta and/or IVOL anomalies include Boyer et al. (2010), Bali et al. (2016), and Schneider et al. (2016). Stambaugh et al. (2015) observe that high-IVOL stocks indeed tend to have substantially higher positive skewness compared to low-IVOL stocks but that this difference is very similar among both underpriced and overpriced stocks. In contrast, the alpha-IVOL relation is positive among underpriced stocks but negative among overpriced stocks. A similar challenge would seem to arise for skewness-based explanations of the beta anomaly. It is not clear why such explanations would apply only within overpriced stocks.

Table 3
Alphas on portfolios formed by sorting on mispricing score and beta; deleting overpriced high-IVOL stocks.

The table reports alphas for portfolios formed by sorting independently on mispricing scores and pre-ranking betas after deleting about 7% of the stock universe: stocks in both the top mispricing quintile (i.e., most overpriced) and the top quartile of IVOL. Alphas are computed with respect to the three factors of Fama and French (1993). A stock's mispricing score, following Stambaugh et al. (2015), is its average ranking with respect to 11 prominent return anomalies. A stock's pre-ranking beta, based on a rolling five-year window, is estimated by regressing the stock's monthly return on the contemporaneous market return plus lagged monthly return, summing the slope coefficients, and then applying shrinkage. The sample period is from January 1963 through December 2013. All *t*-statistics (in parentheses) are based on the heteroskedasticity-consistent standard errors of White (1980).

Mispricing quintile	Beta decile										Highest - Lowest
	Lowest	2	3	4	5	6	7	8	9	Highest	
Underpriced	0.24	0.36	0.36	0.40	0.32	0.31	0.31	0.37	0.29	0.46	0.22
	(2.22)	(3.36)	(3.38)	(3.55)	(2.70)	(2.75)	(2.61)	(2.75)	(1.92)	(2.86)	(1.07)
2	0.30	0.24	−0.01	0.20	0.03	0.16	0.20	−0.07	−0.04	0.04	−0.26
	(2.64)	(2.23)	(−0.04)	(1.68)	(0.28)	(1.27)	(1.55)	(−0.55)	(−0.26)	(0.24)	(−1.24)
3	0.12	0.03	0.06	0.04	−0.18	0.17	−0.09	−0.19	0.01	0.08	−0.04
	(1.09)	(0.25)	(0.49)	(0.32)	(−1.45)	(1.17)	(−0.80)	(−1.41)	(0.04)	(0.52)	(−0.18)
4	−0.17	−0.26	−0.13	−0.22	−0.27	−0.22	−0.12	−0.48	−0.22	−0.14	0.03
	(−1.31)	(−2.04)	(−1.13)	(−1.76)	(−2.07)	(−1.59)	(−0.86)	(−3.42)	(−1.48)	(−0.96)	(0.12)
Overpriced	−0.20	−0.23	−0.19	−0.51	−0.39	−0.45	−0.74	−0.35	−0.62	−0.56	−0.36
	(−1.54)	(−1.74)	(−1.50)	(−3.84)	(−2.72)	(−3.54)	(−4.64)	(−2.42)	(−4.15)	(−3.58)	(−1.78)
All stocks	0.11	0.13	0.13	0.05	−0.11	0.02	−0.07	−0.06	−0.07	−0.11	−0.23
	(1.55)	(1.70)	(1.94)	(0.71)	(−1.65)	(0.22)	(−1.05)	(−0.72)	(−0.75)	(−1.08)	(−1.52)

Table 4
Alphas on portfolios formed by sorting on mispricing score and beta; deleting high−beta stocks.

The table reports alphas for portfolios formed by sorting independently on mispricing scores and pre-ranking betas after deleting stocks with pre-ranking betas in the top 7%. Alphas are computed with respect to the three factors of Fama and French (1993). A stock's mispricing score, following Stambaugh et al. (2015), is its average ranking with respect to 11 prominent return anomalies. A stock's pre-ranking beta, based on a rolling five-year window, is estimated by regressing the stock's monthly return on the contemporaneous market return plus lagged monthly return, summing the slope coefficients, and then applying shrinkage. The sample period is from January 1963 through December 2013. All *t*-statistics (in parentheses) are based on the heteroskedasticity-consistent standard errors of White (1980).

Mispricing quintile	Beta decile										Highest - Lowest
	Lowest	2	3	4	5	6	7	8	9	Highest	
Underpriced	0.28	0.36	0.36	0.37	0.42	0.20	0.24	0.54	0.36	0.29	0.02
	(2.50)	(3.28)	(3.42)	(3.41)	(3.72)	(1.69)	(1.90)	(3.96)	(2.43)	(1.90)	(0.10)
2	0.30	0.21	0.05	0.07	0.13	0.11	0.03	0.00	−0.12	−0.03	−0.33
	(2.67)	(1.83)	(0.49)	(0.69)	(1.13)	(0.98)	(0.26)	(0.03)	(−0.92)	(−0.26)	(−1.79)
3	0.11	−0.04	−0.02	0.05	−0.13	0.02	0.10	−0.22	0.05	−0.10	−0.21
	(1.03)	(−0.27)	(−0.17)	(0.41)	(−1.02)	(0.15)	(0.75)	(−1.73)	(0.33)	(−0.69)	(−1.13)
4	−0.03	−0.28	−0.05	−0.19	−0.32	−0.48	−0.09	−0.19	−0.35	−0.44	−0.42
	(−0.20)	(−2.27)	(−0.43)	(−1.48)	(−2.74)	(−3.57)	(−0.61)	(−1.24)	(−2.69)	(−3.38)	(−2.12)
Overpriced	−0.28	−0.30	−0.28	−0.41	−0.61	−0.69	−0.70	−0.63	−0.92	−0.93	−0.65
	(−2.12)	(−2.04)	(−2.00)	(−2.78)	(−3.81)	(−4.75)	(−4.16)	(−4.10)	(−6.42)	(−5.78)	(−3.27)
All stocks	0.12	0.12	0.15	0.04	−0.03	−0.09	−0.08	−0.02	−0.15	−0.23	−0.35
	(1.57)	(1.52)	(2.20)	(0.58)	(−0.43)	(−1.31)	(−0.99)	(−0.29)	(−1.79)	(−2.47)	(−2.52)

3.3. Evidence of IVOL's role

The importance of IVOL in generating the beta anomaly can be demonstrated in a number of ways. We first simply eliminate stocks in the intersection of the highest 20% of the mispricing measure and the highest 25% of IVOL. On average, the number of stocks eliminated is 7% of those in our universe, representing only 1% of total market value. Table 3 repeats the analysis in Table 2 for the remaining stocks. We see that eliminating just 7% of the stocks is sufficient to render the beta anomaly insignificant. The bottom right cell equals −23 bps, one-fourth less than the corresponding value in Table 2, and the *t*-statistic is only −1.52. The 8 bps difference in the two results has a *t*-statistic of 5.75. In other words, the significant beta anomaly in the overall universe is sensitive to the presence of overpriced stocks with high IVOL.

Suppose that beta is the characteristic driving the beta anomaly. Then eliminating the 7% of stocks having the highest betas should presumably reduce the significance of the beta anomaly at least as much as eliminating 7% by other criteria. Table 4 reports the results of eliminating these high-beta stocks (representing 5% of our universe's market value) and again repeating the analysis in Table 2. Unlike the result in Table 3, the bottom right cell of Table 4 reveals a beta anomaly of −35 bps with a *t*-statistic of −2.52, actually a bit stronger than the Table 2 result of -31 bps with a *t*-statistic of −2.08. This result in Table 4, when compared to the insignificant beta anomaly in Table 3, seems inconsistent with beta driving the beta anomaly. The beta anomaly remains strong in Table 4 despite the fact that the range of betas is reduced there substantially more than in Table 3. Eliminating the high-beta stocks reduces the post-ranking beta

Table 5
Alphas on portfolios formed by sorting on beta and IVOL.
The table reports alphas for portfolios formed by sorting independently on IVOL and pre-ranking betas. Alphas are computed with respect to the three factors of Fama and French (1993). A stock's mispricing score, following Stambaugh et al. (2015), is its average ranking with respect to 11 prominent return anomalies. A stock's pre-ranking beta, based on a rolling five-year window, is estimated by regressing the stock's monthly return on the contemporaneous market return plus one lagged monthly return, summing the slope coefficients, and then applying shrinkage. IVOL is computed as the standard deviation of the most recent month's residuals in a regression of each stock's daily return on daily realizations of the three Fama-French factors. The last column, labeled "Average," reports the average across the ten beta deciles; similarly, the last row of cells reports the average across the five mispricing quintiles. The sample period is from January 1963 through December 2013. All t-statistics (in parentheses) are based on the heteroskedasticity-consistent standard errors of White (1980).

IVOL quintile	Beta decile										Highest - Lowest	Average
	Lowest	2	3	4	5	6	7	8	9	Highest		
Lowest	0.10	0.12	0.16	0.06	−0.19	0.05	−0.08	−0.03	−0.02	0.20	0.10	0.04
	(1.14)	(1.33)	(1.71)	(0.55)	(−1.79)	(0.40)	(−0.71)	(−0.23)	(−0.09)	(1.21)	(0.54)	(0.67)
2	0.15	0.16	0.07	0.04	0.00	−0.19	−0.02	−0.10	−0.10	−0.17	−0.31	−0.02
	(1.36)	(1.46)	(0.68)	(0.38)	(−0.04)	(−1.69)	(−0.13)	(−0.78)	(−0.79)	(−1.20)	(−1.68)	(−0.33)
3	0.07	0.10	0.04	−0.15	−0.02	0.15	0.03	−0.02	0.05	−0.06	−0.12	0.02
	(0.48)	(0.79)	(0.34)	(−1.16)	(−0.13)	(1.16)	(0.23)	(−0.16)	(0.33)	(−0.41)	(−0.61)	(0.39)
4	0.09	0.04	0.07	0.17	0.03	0.05	−0.11	−0.07	−0.33	−0.17	−0.28	−0.02
	(0.48)	(0.25)	(0.46)	(1.17)	(0.23)	(0.35)	(−0.69)	(−0.53)	(−2.44)	(−1.15)	(−1.10)	(−0.35)
Highest	−0.66	−0.24	−0.61	−0.38	−0.35	−0.12	−0.72	−0.99	−0.56	−0.85	−0.22	−0.55
	(−2.40)	(−1.08)	(−2.84)	(−2.11)	(−1.92)	(−0.63)	(−3.75)	(−5.30)	(−3.08)	(−4.80)	(−0.65)	(−6.46)
Highest −	−0.77	−0.35	−0.77	−0.44	−0.16	−0.17	−0.64	−0.96	−0.54	−1.05	−0.27	−0.58
Lowest	(−2.81)	(−1.50)	(−3.27)	(−2.10)	(−0.78)	(−0.74)	(−2.93)	(−4.16)	(−2.32)	(−4.69)	(−0.74)	(−5.46)
Average	−0.05	0.04	−0.05	−0.05	−0.11	−0.01	−0.18	−0.24	−0.19	−0.21	−0.16	
	(−0.49)	(0.44)	(−0.71)	(−0.80)	(−1.68)	(−0.17)	(−2.71)	(−3.38)	(−2.18)	(−2.14)	(−1.02)	

difference between the highest and lowest beta deciles to 0.80, versus 0.92 for the total universe. In contrast, that difference is reduced less than one-sixth as much, just to 0.90, by eliminating the overpriced high-IVOL stocks.

The importance of IVOL to the beta anomaly is also revealed by a double-sort on IVOL and beta. Each month we independently assign stocks to beta deciles and IVOL quintiles, and then we construct value-weighted portfolios in each of the 10×5 intersecting cells. Table 5 reports the alpha on each portfolio, the high-low alpha difference for a given variable within each level of the other variable, and the average of those high-low differences across all levels of the other variable. Four of the five high-low beta spreads are negative, but only one is even marginally significant: the second-lowest IVOL quintile produces an alpha spread of −31 bps with a t-statistic of −1.68. Moreover, the high-low beta spread averaged across all IVOL quintiles is just −16 bps with a t-statistic of −1.02. Overall, there is little evidence of a beta anomaly once one controls for IVOL.

In contrast, the overall negative alpha-IVOL relation remains strong after controlling for beta. The high-low IVOL spread produces a negative alpha in all beta deciles, significantly so in seven of the ten. In addition, the IVOL spread's alpha averaged across the beta deciles is −58 bps with a t-statistic of −5.46.

We also take a somewhat more parametric approach to control for IVOL in order to re-examine the beta anomaly within each mispricing quintile. Each month, we estimate the regression,

$$z(\hat{\beta}_{i,t}) = \sum_{j=1}^{5} I(M_{i,t} = j)(a_j + b_j z(IVOL_{i,t})) + \epsilon_{i,t}, \quad (6)$$

where $z(\hat{\beta}_{i,t})$ and $z(IVOL_{i,\,t})$ are the cross-sectional z-scores corresponding to the beta and IVOL cross-sectional percentiles in month t, and $I(M_{i,t} = j)$ is the indicator function that equals one if stock i falls into mispricing quintile j in month t and zero otherwise.[4] We then define the residual-beta z-score as $\epsilon_{i,\,t}$. Table 6 repeats the analysis reported in Table 2, except that instead of sorting on beta we sort on residual-beta z-score. In other words, we essentially sort on the component of beta that is unrelated to IVOL within each mispricing quintile. Table 6 shows there is no significant beta effect after applying this control for IVOL. In Table 6, the largest negative alpha for the high-low spread in IVOL-adjusted beta occurs in the quintile of most-overpriced stocks, but even there the alpha is just −23 bps with a t-statistic of −1.11. In the overall universe, the alpha for the spread in IVOL-adjusted beta, reported in the bottom-right cell of Table 6, is −16 bps with a t-statistic of −1.09.

The results in Tables 3 through 6 provide direct evidence of IVOL's key role in the beta anomaly. The anomaly does not survive deletion of high-IVOL overpriced stocks, nor does it survive controlling for IVOL either by double-sorting or regression. Before moving on, however, we look for additional evidence of IVOL's role by exploiting variation over time in the beta-IVOL correlation.

3.4. Time-varying beta-IVOL correlation and sentiment

Our proposed explanation of the beta anomaly requires the presence of overpriced stocks as well as a positive correlation between beta and IVOL. Without overpriced stocks, IVOL plays no role in deterring the correction of overpricing, and thus a negative alpha-IVOL relation does not arise. Even when that negative relation arises, it does

[4] We allow a_j and b_j to differ across mispricing segments, because F-tests reject the nulls (with p-values less than 0.001) that these coefficients are equal across mispricing segments.

Table 6
Alphas on portfolios formed by sorting on mispricing score and IVOL-adjusted beta.

The table reports alphas for portfolios formed by sorting independently on mispricing scores and the IVOL-orthogonal component of beta. Alphas are computed with respect to the three factors of Fama and French (1993). A stock's mispricing score, following Stambaugh et al. (2015), is its average ranking with respect to 11 prominent return anomalies. The IVOL-adjusted component of beta for stock i in month t is the residual $\epsilon_{i,t}$ in the cross-sectional regression

$$z(\hat{\beta}_{i,t}) = \sum_{j=1}^{5} I(M_{i,t} = j)(a_j + b_j z(IVOL_{i,t})) + \epsilon_{i,t},$$

where $z(\hat{\beta}_{i,t})$ and $z(IVOL_{i,t})$ are the z-scores of pre-ranking betas and IVOL in the cross-section in month t, and $I(M_{i,t} = j)$ is the indicator function equal to one (zero otherwise) if stock i is in mispricing quintile j in month t. A stock's pre-ranking beta, based on a rolling five-year window, is estimated by regressing the stock's monthly return on the contemporaneous market return plus lagged monthly return, summing the slope coefficients, and then applying shrinkage. IVOL is computed as the standard deviation of the most recent month's residuals in a regression of each stock's daily return on daily realizations of the three Fama-French factors. The sample period is from January 1963 through December 2013. All t-statistics (in parentheses) are based on the heteroskedasticity-consistent standard errors of White (1980).

Mispricing quintile	Beta decile										Highest - Lowest
	Lowest	2	3	4	5	6	7	8	9	Highest	
Underpriced	0.22	0.34	0.20	0.34	0.37	0.34	0.29	0.36	0.22	0.31	0.09
	(1.87)	(3.19)	(1.75)	(2.93)	(3.09)	(3.15)	(2.53)	(3.13)	(1.68)	(2.31)	(0.49)
2	0.24	0.15	0.11	0.02	0.39	0.14	−0.10	0.03	−0.12	0.01	−0.23
	(2.13)	(1.29)	(0.98)	(0.17)	(3.29)	(1.17)	(−0.87)	(0.24)	(−1.02)	(0.10)	(−1.24)
3	0.00	−0.08	0.00	−0.03	−0.07	−0.02	−0.05	0.02	0.05	0.04	0.04
	(−0.01)	(−0.65)	(0.03)	(−0.27)	(−0.55)	(−0.12)	(−0.36)	(0.14)	(0.38)	(0.32)	(0.21)
4	−0.08	−0.21	−0.28	−0.29	−0.34	−0.22	−0.40	−0.21	−0.29	−0.17	−0.10
	(−0.59)	(−1.74)	(−2.24)	(−2.22)	(−2.59)	(−1.57)	(−2.74)	(−1.51)	(−2.31)	(−1.17)	(−0.46)
Overpriced	−0.51	−0.27	−0.42	−0.82	−0.74	−0.58	−0.67	−0.93	−1.05	−0.74	−0.23
	(−4.08)	(−1.92)	(−2.87)	(−5.52)	(−4.01)	(−3.99)	(−4.20)	(−5.81)	(−6.10)	(−4.44)	(−1.11)
All stocks	0.07	0.11	0.03	0.04	0.03	0.01	−0.09	−0.03	−0.15	−0.09	−0.16
	(0.85)	(1.42)	(0.37)	(0.52)	(0.41)	(0.16)	(−1.33)	(−0.42)	(−1.94)	(−0.97)	(−1.09)

not produce the beta anomaly without a positive beta-IVOL correlation, especially within the overpriced stocks. Put differently, the conditions most conducive to the beta anomaly are a substantial presence of overpriced stocks coupled with a high beta-IVOL correlation among those stocks.

We pursue this point in conducting a time-series investigation of IVOL's role in the beta anomaly. To identify periods with a substantial presence of overpriced stocks, we use the monthly index of investor sentiment constructed by Baker and Wurgler (2006). When that index is high, indicating investor optimism, we assume overpricing of stocks is more likely, and thus the negative alpha-IVOL relation is stronger. Stambaugh et al. (2015) find that the latter relation is indeed stronger following high sentiment. We also compute each month the correlation between beta and IVOL by standardizing our estimates of both quantities, transforming those standardized estimates into cross-sectional z-scores, and then computing the correlation between the two z-scores within the quintile of the most-overpriced stocks.

Fig. 2 plots the monthly series of sentiment and the beta-IVOL correlation. The series exhibit significant variation but only modest comovement. Sentiment reaches its highest value in the late 1960s and then falls to its lowest trough in the 1970s. In contrast, the beta-IVOL correlation hits a significant trough near zero in the late 1960s and reaches its highest values in the early and mid-1970s. The beta-IVOL correlation is again nearly zero in the mid-90s and late 2000s, both periods in which sentiment is about average. On the other hand, both series experience relative peaks in the early 1980s and early 2000s. We next exploit the fact that there are some periods when both series are high but other periods when one or both are not.

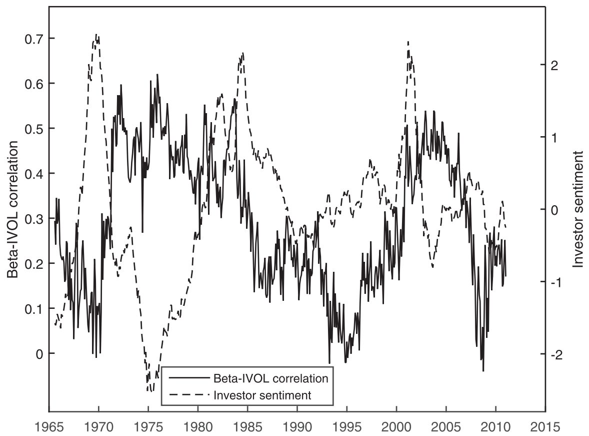

Fig. 2. Beta-IVOL correlation and investor sentiment. The figure plots the monthly time series of the cross-sectional correlation between beta and IVOL within the most-overpriced quintile (solid line) and the Baker and Wurgler (2006) investor sentiment index (dashed line). The sample period covers January 1965 through January 2011.

We assign the months from 1965 through 2010 to four regimes: high correlation and high sentiment (HcHs), low correlation and high sentiment (LcHs), high correlation and low sentiment (HcLs), and low correlation and low sentiment (LcLs). A given month is classified as high (low) sentiment if the previous month's index value is above (below) the whole-sample median; high- and low-correlation months are classified in the same manner. The four regimes reflect the intersection of these two-way classifications. The number of months in each regime is fairly similar across regimes, with HcHs and LcLs having

Table 7
The beta anomaly in periods of high and low levels of investor sentiment and beta-IVOL correlation.

The table reports alphas on value-weighted portfolios containing stocks in the highest and lowest beta deciles. The alphas on the low-beta portfolio, α_L, and the high-beta portfolio, α_H, are computed in each of four regimes. Months are assigned to regimes according to whether investor sentiment and the most-overpriced stocks' beta-IVOL correlation are above or below their median values. Alphas are estimated in the regression

$$R_{i,t} = \sum_{j=1}^{4} \alpha_j D_{j,t} + \delta_1 MKT_t + \delta_2 SMB_t + \delta_3 HML_t + \epsilon_{i,t}, \quad (18)$$

where $R_{i,\,t}$ is the return on the high-beta decile portfolio, the return on the low-beta decile portfolio, or the difference in those returns (high minus low). The regime dummy $D_{j,\,t}$ equals one if month t is in regime j and zero otherwise, α_j is the alpha in regime j, and MKT_t, SMB_t, and HML_t are the three factors defined by Fama and French (1993). The sample period is from August 1965 through January 2011. All t-statistics (in parentheses) are based on the heteroskedasticity-consistent standard errors of White (1980). The F-statistic tests equality across regimes of $\alpha_H - \alpha_L$.

Beta-IVOL correlation	Investor sentiment	α_L	α_H	$\alpha_H - \alpha_L$	Months in regime
High	High	0.48	−0.68	−1.16	112
		(2.27)	(−2.44)	(−2.91)	
Low	High	−0.01	−0.13	−0.12	160
		(−0.10)	(−0.71)	(−0.43)	
High	Low	0.12	−0.23	−0.35	161
		(0.98)	(−1.11)	(−1.23)	
Low	Low	−0.17	0.33	0.51	113
		(−1.05)	(1.32)	(1.35)	
F-statistic:				3.73	
(p-value:)				(0.01)	

somewhat fewer months, 112 and 113, respectively, compared to 160 and 161 for each of LcHs and HcLs.[5]

Table 7 reports alphas for the high-low beta spreads in each of the four regimes. The alphas are estimated as coefficients on regime dummy variables in the regression

$$R_{H,t} - R_{L,t} = \sum_{j=1}^{4} \alpha_j D_{j,t} + \delta_1 MKT_t + \delta_2 SMB_t + \delta_3 HML_t + \epsilon_{i,t}, \quad (7)$$

where $R_{H,\,t}$ and $R_{L,\,t}$ are the returns on the high- and low-beta decile portfolios in month t, $D_{j,\,t}$ equals one if month t is in regime j and zero otherwise, α_j is the alpha in regime j, and MKT_t, SMB_t, and HML_t are the three factors defined by Fama and French (1993).

Only the high-correlation/high-sentiment regime, HcHs, exhibits a significant alpha on the high-low beta spread, consistent with a high beta-IVOL correlation and a substantial presence of overpricing being the conditions most conducive to the beta anomaly. In that regime, the monthly alpha is −116 bps with a t-statistic of −2.91. The other negative alphas occur in the LcHs regime and HcLs regime, where the level of one or the other of the two series is high, but those alphas are substantially smaller: the largest in magnitude is −35 bps with a t-statistic of just −1.23. In the regime with both low beta-IVOL correlation and low sentiment, the alpha is actually positive and thus opposite the beta anomaly, though the value is just 51 bps with a t-statistic of 1.35. An F-test of equality of alphas across the four regimes produces a p-value of 0.01. Overall, the results of this investigation exploiting variation in sentiment and the beta-IVOL correlation are consistent with our explanation of IVOL's role in producing the beta anomaly.

Antiniou et al. (2016) and Shen et al. (2017) also propose sentiment-related explanations in which the beta anomaly is stronger when sentiment is high. Their explanations, different from ours, do not involve IVOL or the IVOL-beta correlation. The results in Table 7 are useful in judging both studies' explanations relative to ours. We see that high sentiment alone is not sufficient to generate the beta anomaly: periods with high sentiment but low beta-IVOL correlation exhibit no beta anomaly.

4. Betting against beta?

Frazzini and Pedersen (2014) analyze a betting-against-beta (BAB) strategy designed to exploit the beta anomaly. The BAB strategy goes long a portfolio of low-beta stocks and short a portfolio of high-beta stocks, taking a larger long position than short position so that the overall strategy has a zero beta. The strategy is financed with riskless borrowing, so

$$r_{t+1}^{BAB} = \frac{1}{\beta^L}(r_{t+1}^L - r^f) - \frac{1}{\beta^H}(r_{t+1}^H - r^f) \quad (8)$$

is the payoff on this zero-investment strategy having long and short positions of sizes $1/\beta^L$ and $1/\beta^H$, where β^L and β^H are the betas on the long and short portfolios. Each of those portfolios is constructed using individual-stock beta rankings to determine weights. Specifically, if r_{t+1} denotes the vector of returns on the n individual stocks in the trading universe, then $r_{t+1}^L = r_{t+1}'\omega_L$ and $r_{t+1}^H = r_{t+1}'\omega_H$, where $\omega_H = k(z - \bar{z})^+$, $\omega_L = k(z - \bar{z})^-$, z is an n-vector with ith element equal to $z_{it} = rank(\beta_{it})$, β_{it} is the estimated beta for stock i, $\bar{z}$ is the average z_{it}, x^+ and x^- denote the positive

[5] Observations equal to the median are assigned to the low regime.

Table 8
Sources of betting-against-beta profits.
The table reports the components of the betting-against-beta (BAB) alpha, α_{BAB}, which is decomposed as

$$\alpha_{BAB} = (\alpha_L - \alpha_H) + \left[\left(\frac{1}{\beta_L} - 1\right)\alpha_L + \left(1 - \frac{1}{\beta_H}\right)\alpha_H\right],$$

where α_L and α_H are the alphas of the low- and high-beta portfolios, and $\frac{1}{\beta_L}$ and $\frac{1}{\beta_H}$ are average reciprocals of the long- and short-leg betas. Alphas are computed with respect to the three-factor model of Fama and French (1993), and t-statistics are reported in parentheses. Results are shown within each quintile of the mispricing measure as well as for the total stock universe. The sample period is January 1963 through December 2013.

Mispricing quintile	α_L	α_H	$1/\beta_L$	$1/\beta_H$	$\alpha_L - \alpha_H$	$(1/\beta_L - 1)\alpha_L$ $+(1 - 1/\beta_H)\alpha_H$	α_{BAB}
Underpriced	0.49	0.52	1.51	0.74	−0.03	0.41	0.38
	(8.62)	(8.69)			(−0.37)	(5.42)	(3.80)
2	0.31	0.24	1.58	0.72	0.08	0.31	0.39
	(5.00)	(3.93)			(0.89)	(3.65)	(3.25)
3	0.12	0.10	1.64	0.72	0.02	0.25	0.27
	(2.18)	(1.61)			(0.25)	(2.85)	(2.31)
4	−0.11	−0.18	1.61	0.71	0.08	0.10	0.18
	(−1.69)	(−2.54)			(0.77)	(1.07)	(1.43)
Overpriced	−0.47	−0.88	1.47	0.70	0.41	−0.33	0.08
	(−5.89)	(−8.04)			(3.41)	(−3.17)	(0.67)
All stocks	0.12	−0.08	1.56	0.71	0.20	0.16	0.36
	(2.49)	(−1.28)			(2.40)	(1.93)	(3.47)

and negative elements of a vector x, and k is a normalizing constant such that the elements of both ω_H and ω_L sum to one.

As Frazzini and Pedersen (2014) document, the BAB strategy produces significant profits across a variety of asset markets. We re-examine its performance in the US stock market along two dimensions. First, in Section 4.1, we look at the extent to which the strategy's profitability is attributable to exploiting the beta anomaly versus taking a levered net-long position in mispriced stocks. Second, motivated by our IVOL-based explanation of the beta anomaly, we explore in Section 4.2 whether a betting-against-IVOL spread subsumes the profitability of the BAB spread.

4.1. Sources of BAB alpha

From Eq. (8), the alpha for the BAB strategy can be decomposed as

$$\begin{aligned}\alpha_{BAB} &= \frac{1}{\beta^L}\alpha_L - \frac{1}{\beta^H_t}\alpha_H \\ &= (\alpha_L - \alpha_H) + \left[\left(\frac{1}{\beta_L} - 1\right)\alpha_L + \left(1 - \frac{1}{\beta_H}\right)\alpha_H\right], \qquad (9)\end{aligned}$$

where α_L and α_H are the alphas on the high- and low-beta portfolios. The first term on the right-hand side of Eq. (9), $(\alpha_L - \alpha_H)$, is the alpha on the beta spread. That is, this component of α_{BAB} reflects the beta anomaly examined above. The second term, in square brackets, adds α_L and α_H, with each multiplied by positive coefficients, given $\beta_L < 1 < \beta_H$. This component of α_{BAB} is not directly related to the beta anomaly, given that both α_L and α_H receive positive weights. Essentially, this component simply reflects the fact that the BAB strategy is overall a levered net-long position, given the larger size of the long position versus the short. This second component can nevertheless be a source of profit. For example, if $\alpha_L = \alpha_H = \bar{\alpha} > 0$, so that both the high- and low-beta portfolios have positive alpha that is unrelated to beta, then this second component of α_{BAB} is the positive quantity $(1/\beta_L - 1/\beta_H)\bar{\alpha}$.

We compute the BAB alpha for our total universe as well as for each of the mispricing quintiles, applying the decomposition in Eq. (9) in each case. Table 8 reports the results. The last column contains the BAB strategy's alpha, α_{BAB}, and the preceding columns contain the quantities appearing in the decomposition of α_{BAB} in (9). In the total universe, α_{BAB} equals 36 bps per month, with a t-statistic of 3.47. More than half of that alpha, 20 bps (t-statistic: 2.40), is contributed by the first term in (9) that reflects the beta anomaly. The other component, reflecting the strategy's overall levered net-long position, is a nontrivial 16 bps (t-statistic: 1.93). In other words, a significant portion of the profit from a BAB strategy need not stem from the beta anomaly.

This point emerges even more sharply from the results in Table 8 for the separate mispricing quintiles. The three least overpriced quintiles produce economically and statistically significant BAB profit, with α_{BAB} ranging between 27 and 39 bps per month and t-statistics between 2.31 and 3.80. Strikingly, the most-overpriced quintile, in which the beta anomaly is far stronger than in the other four, yields an α_{BAB} of 8 bps with a t-statistic of just 0.67. We see from Table 8 that the BAB profits in the other four quintiles owe much to the second term in (9), which accounts for between 56% and 108% of their α_{BAB} values. For example, in the quintile of most-underpriced stocks, where both α_L and α_H are (not surprisingly) significantly positive, that second component of α_{BAB} equals 41 bps, more than the overall α_{BAB} in the total universe. The contribution of $(\alpha_L - \alpha_H)$ in that quintile is negative 3 bps, reflecting the absence of a significant beta anomaly among the underpriced stocks.

The fact that the BAB strategy produces the smallest alpha among the stocks exhibiting by far the strongest beta anomaly, the most-overpriced stocks, further underscores the importance of both components in Eq. (9). In that quintile we see a strong contribution of 41 bps by $(\alpha_L - \alpha_H)$, reflecting the beta anomaly, but most of that contribution to α_{BAB} is offset by the second component, equal to −33 bps, reflecting the negative values of both α_L and α_H associated with overpricing. In other words, the BAB strategy's ability to exploit the beta anomaly where it exists most strongly is foiled by the strategy's levered net-long position in overpriced stocks.

The first component in Eq. (9) is the alpha on what might reasonably be termed the "unlevered" BAB strategy. That strategy, also zero-investment, directly exploits the beta anomaly but does not employ leverage in order to achieve a zero beta. This unlevered BAB strategy, which yields an alpha of 41 bps (*t*-statistic: 3.41) in the quintile of most-overpriced stocks, as reported in Table 8, delivers an alpha of just 6 bps (*t*-statistic: 0.79) in the remaining portion of the stock universe. Here again we see that the beta anomaly is much stronger among the overpriced stocks. The difference between this result and the spreads between the beta-ranked portfolios examined in Table 2 is simply that the latter analysis compares value-weighted portfolios in the extreme beta deciles, whereas here we compare beta-weighted portfolios of stocks in the two halves of the beta distribution.

4.2. BAB versus betting against IVOL

Frazzini and Pedersen (2014) examine the robustness of BAB profits to controlling for IVOL by constructing a BAB strategy within each IVOL decile. They find significant BAB profits within each decile. Given our previous discussion, however, significant BAB profits need not reflect a beta anomaly. For example, with a relation between alpha and IVOL, the alphas on both the high- and low-beta portfolios in a given IVOL decile can equal the same positive value if there is no beta anomaly within that decile. In that case the first term in Eq. (9) equals zero, but the second term nevertheless delivers a positive BAB profit. In other words, even if BAB profits are robust to controlling for IVOL, the beta anomaly need not be.

In addition to the approaches we take in Section 3 to control for IVOL when assessing the beta anomaly, here we explore yet another. We ask whether the unlevered BAB strategy discussed above produces an alpha with respect to a set of factors that include unlevered "betting-against-IVOL" (BAI) strategies constructed analogously to the unlevered BAB strategy. Recall that the direction of the relation between alpha and IVOL depends on the direction of mispricing. We therefore first construct two BAI strategies, one within the quintile of the most-underpriced stocks and the other within the most-overpriced quintile. For each strategy, we follow the same procedure detailed after Eq. (8) for the BAB strategy, with just two departures. First, $z_{it} = rank(\sigma_{it})$, where σ_{it} is the estimated IVOL for stock *i*, and, second, $\bar{z}$ is the average z_{it} within the given mispricing quintile. For the overpriced stocks, the long and short legs of the unlevered BAI strategy are otherwise identified and weighted identically as in the unlevered BAB strategy, consistent with the negative alpha-IVOL relation among overpriced stocks. For the underpriced stocks, the roles of long and short are reversed, given the positive alpha-IVOL relation within that segment.

The unlevered BAI strategy for the overpriced stocks has an alpha of 105.1 bps (*t*-statistic: 9.78), and the strategy's alpha for underpriced stocks is 22.90 bps (*t*-statistic: 2.72). These results echo those of Stambaugh et al. (2015), who find a significantly positive alpha-IVOL relation among underpriced stocks but an even stronger negative relation among overpriced stocks. As before, alphas are computed with respect to the three factors of Fama and French (1993). A simple average of the return spreads on the overpriced and underpriced BAI strategies yields an alpha of 64 bps (*t*-statistic: 11.22). It also happens that the simple market beta of this combination BAI strategy is nearly zero (−0.05).

Recall from the last row of Table 8 that the unlevered BAB strategy in the total universe has a monthly alpha of 20 bps (*t*-statistic: 2.40) with respect to the three Fama-French factors. If those factors are augmented by an additional factor, the average of the underpriced and overpriced BAI series, the BAB alpha becomes −8 bps (*t*-statistic: −0.73). That is, the beta anomaly, when exploited by the unlevered BAB strategy, does not survive this control for IVOL. In contrast, the averaged BAI strategy, which is essentially zero-beta, produces a monthly alpha of 60 bps (*t*-statistic: 10.77) with respect to the three Fama–French factors plus the BAB series.

5. Conclusions

We provide an explanation for the beta anomaly, which is negative (positive) alpha on stocks with high (low) beta. The anomaly arises from beta's positive cross-sectional correlation with IVOL. As shown by Stambaugh et al. (2015), the relation between alpha and IVOL is positive among underpriced stocks but negative and stronger among overpriced stocks, where mispricing is gauged by a multi-anomaly measure. This mispricing-dependent direction of the alpha-IVOL relation is consistent with IVOL reflecting risk that deters arbitrage-driven price correction. The stronger negative relation among overpriced stocks is consistent with a lower amount of capital being able or willing to bear the risks of shorting overpriced stocks as compared to the amount of capital available for buying underpriced stocks. The asymmetry produces a negative alpha-IVOL relation in the total stock universe. This negative alpha-IVOL relation combines with the positive beta-IVOL correlation to produce a significantly negative alpha-beta relation, the beta anomaly.

Consistent with this explanation, a significant beta anomaly appears only among overpriced stocks. Also consistent with our explanation, the beta anomaly does not survive various controls for IVOL, and excluding just 7% of the stock universe, overpriced stocks with high IVOL, renders the beta anomaly insignificant.

Our explanation of the beta anomaly requires a substantial presence of overpriced stocks coupled with a positive beta-IVOL correlation. We should therefore expect the strongest beta anomaly in periods when overpricing is especially likely and the beta-IVOL correlation among the most-overpriced stocks is especially high. The data support this prediction when we use high levels of investor sentiment to proxy for periods when overpricing is most likely. We find a significant beta anomaly in periods when investor sentiment and the beta-IVOL correlation are both above their median values but not when either or both quantities are below their medians.

The Frazzini and Pedersen (2014) betting-against-beta (BAB) strategy, which is levered to achieve a zero beta, has one source of profit that exploits the beta anomaly, but it has an additional source of potential profit reflecting its levered net-long position in stocks that may have positive alphas for reasons unrelated to the beta anomaly. An unlevered version of the BAB strategy that reflects a direct play on the beta anomaly does not produce a significant alpha with respect to factors that include analogously constructed betting-against-IVOL (BAI) return. In contrast, the BAI strategy produces a large alpha with respect to factors that include the BAB return.

Appendix

A.1. Comparing beta-estimation methods

For each beta-sorted portfolio i in month t, define the out-of-sample "hedging error" as

$$h_{i,t} = R_{i,t} - \hat{\beta}_{i,t} R_{m,t}, \quad \text{(A.1)}$$

Table A.1
Comparing beta-estimation methods.
The table compares our beta-estimation method, which uses five years of monthly returns with a one-lag Dimson (1979) correction and Vasicek (1973) shrinkage ("monthly 5-year shrunk"), to four other estimation methods from the beta-anomaly literature: one year of daily returns with a five-lag Dimson correction ("daily 1-year"), the former with the three least recent lags constrained to have the same coefficient ("daily 1-year constrained"), five years of monthly returns with a one-lag Dimson correction ("monthly 5-year"), and the method of Frazzini and Pedersen (2014) that separately estimates correlations and volatilities ("Frazzini-Pedersen"). For a given beta-estimation method we compute each stock's out-of-sample "hedging error" in each month t, the difference between the stock's return and the stock's estimated beta times the market return, with the estimation window for beta ending in month $t-1$. We compute the value-weighted (i.e., portfolio-level) average of these hedging errors across all stocks in the same beta decile as of the end of month $t-1$. The table reports the ratio of the variance of this portfolio-level hedging error to the variance of the market return (averaged over rolling five-year windows). We form beta deciles five different ways, using each of the estimation methods, giving 50 portfolios in total. For example, in Panel A, the hedged return of each portfolio is constructed using our beta-estimation method. Each row of the panel indicates which beta-estimation is used to sort stocks in forming the decile portfolios. The last column gives the average value of decile 1 and decile 10. The last row in each panel ("Average") contains the average of the five values displayed in the five rows above. The sample period is from 1963/1 to 2013/12.

Estimation method for	Beta decile										Avg of
forming deciles	Lowest(L)	2	3	4	5	6	7	8	9	Highest (H)	H & L
Panel A: Monthly 5-year shrunk											
Monthly 5-year shrunk	0.260	0.181	0.141	0.147	0.116	0.126	0.142	0.184	0.242	0.403	0.332
Daily 1-year	0.331	0.236	0.187	0.165	0.138	0.134	0.153	0.191	0.366	0.910	0.620
Daily 1-year constrained	0.356	0.241	0.193	0.173	0.134	0.146	0.150	0.200	0.302	0.896	0.626
Monthly 5-year	0.277	0.187	0.150	0.140	0.122	0.119	0.143	0.214	0.359	0.815	0.546
Frazzini-Pedersen	0.418	0.260	0.206	0.188	0.165	0.159	0.164	0.187	0.251	0.608	0.513
Average	0.328	0.221	0.175	0.162	0.135	0.137	0.150	0.195	0.304	0.726	0.527
Panel B: Daily 1-year											
Monthly 5-year shrunk	0.224	0.159	0.129	0.135	0.122	0.125	0.142	0.179	0.230	0.373	0.299
Daily 1-year	0.526	0.258	0.188	0.156	0.123	0.132	0.151	0.204	0.380	1.029	0.778
Daily 1-year constrained	0.502	0.271	0.198	0.153	0.116	0.131	0.149	0.212	0.314	0.898	0.700
Monthly 5-year	0.238	0.168	0.135	0.129	0.123	0.113	0.136	0.207	0.354	0.779	0.509
Frazzini-Pedersen	0.407	0.238	0.176	0.163	0.149	0.133	0.135	0.173	0.242	0.559	0.483
Average	0.379	0.219	0.165	0.147	0.127	0.127	0.142	0.195	0.304	0.728	0.554
Panel C: Daily 1-year constrained											
Monthly 5-year shrunk	0.222	0.158	0.129	0.135	0.122	0.124	0.141	0.180	0.229	0.369	0.296
Daily 1-year	0.460	0.241	0.178	0.150	0.121	0.130	0.149	0.194	0.363	0.938	0.699
Daily 1-year Constrained	0.519	0.277	0.201	0.152	0.115	0.131	0.148	0.211	0.313	0.893	0.706
Monthly 5-year	0.237	0.165	0.137	0.129	0.122	0.112	0.136	0.205	0.350	0.766	0.502
Frazzini-Pedersen	0.413	0.240	0.177	0.163	0.149	0.134	0.136	0.172	0.241	0.555	0.484
Average	0.370	0.216	0.164	0.146	0.126	0.126	0.142	0.192	0.299	0.704	0.537
Panel D: Monthly 5-Year											
Monthly 5-year shrunk	0.315	0.197	0.142	0.154	0.118	0.135	0.175	0.255	0.389	0.810	0.562
Daily 1-year	0.322	0.229	0.181	0.163	0.141	0.137	0.159	0.184	0.354	0.823	0.572
Daily 1-year constrained	0.345	0.233	0.187	0.167	0.134	0.147	0.152	0.200	0.298	0.772	0.559
Monthly 5-year	0.387	0.217	0.151	0.144	0.127	0.139	0.186	0.315	0.611	1.689	1.038
Frazzini-Pedersen	0.402	0.251	0.191	0.182	0.162	0.161	0.168	0.182	0.249	0.573	0.487
Average	0.354	0.225	0.170	0.162	0.136	0.144	0.168	0.227	0.380	0.933	0.644
Panel E: Frazzini-Pedersen											
Monthly 5-year shrunk	0.291	0.211	0.164	0.152	0.122	0.133	0.148	0.199	0.256	0.427	0.359
Daily 1-year	0.368	0.274	0.206	0.176	0.148	0.144	0.159	0.203	0.360	0.877	0.623
Daily 1-year constrained	0.387	0.273	0.215	0.186	0.143	0.153	0.158	0.218	0.307	0.838	0.613
Monthly 5-year	0.294	0.221	0.183	0.147	0.127	0.123	0.154	0.231	0.379	0.868	0.581
Frazzini-Pedersen	0.424	0.268	0.219	0.190	0.174	0.166	0.178	0.198	0.252	0.579	0.502
Average	0.353	0.249	0.197	0.170	0.143	0.144	0.159	0.210	0.311	0.718	0.535

Table A.2
Cederburg-O'Doherty estimates of alphas on portfolios formed by sorting on mispricing score and beta.

The table reports alphas for portfolios formed by sorting independently on mispricing scores and pre-ranking betas. Alphas are computed with respect to the three factors of Fama and French (1993) following the procedure of Cederburg and O'Doherty (2016), which allows betas on factors to depend on various instruments. Alpha is estimated as the intercept in the regression,

$$ret_{i,t} = \alpha_{i,t} + (\gamma_{m,i,0} + \gamma_{m,i,1}Z^{m}_{i,t-1})MKT_t + (\gamma_{s,i,0} + \gamma_{s,i,1}Z^{s}_{i,t-1})SMB_t + (\gamma_{h,i,0} + \gamma_{h,i,1}Z^{h}_{i,t-1})HML_t + \epsilon_{i,t},$$

where $ret_{i,t}$ is the quarterly return of portfolio i in quarter t, and MKT_t, SMB_t, and HML_t are quarterly returns that compound the Fama-French monthly factors. The instruments in $Z^{m}_{i,t-1}$ include the dividend yield, default premium, and short-term (past 3-month) and long-term (past 36-month) daily market betas. The instruments in $Z^{s}_{i,t-1}$ are the same except that the daily market betas are replaced by daily SMB betas; similarly, in $Z^{h}_{i,t-1}$, daily HML betas are included instead. The sample period is from January 1963 through December 2013. All t-statistics (in parentheses) are based on the heteroskedasticity-consistent standard errors of White (1980).

Mispricing quintile	Beta decile										Highest - Lowest
	Lowest	2	3	4	5	6	7	8	9	Highest	
Underpriced	0.59	0.67	1.18	1.42	0.68	1.27	0.38	1.05	0.15	0.43	−0.16
	(1.69)	(2.33)	(3.58)	(4.37)	(1.81)	(3.84)	(0.90)	(2.19)	(0.43)	(0.96)	(−0.25)
2	0.75	0.12	0.43	0.45	0.14	0.01	0.28	−0.3	0.29	0.54	−0.21
	(2.66)	(0.39)	(1.08)	(1.31)	(0.43)	(0.03)	(0.83)	(−0.84)	(0.71)	(1.36)	(−0.40)
3	0.23	−0.32	−0.37	−0.81	−0.47	0.36	−0.01	−0.65	0.27	−0.31	−0.54
	(0.69)	(−1.14)	(−1.12)	(−2.16)	(−1.21)	(0.87)	(−0.03)	(−1.53)	(0.59)	(−0.67)	(−0.90)
4	−0.19	−0.74	−0.45	−0.65	−0.03	−0.55	−1.24	−1.04	−1.16	−1.21	−1.01
	(−0.43)	(−2.01)	(−1.39)	(−1.72)	(−0.07)	(−1.26)	(−2.93)	(−2.43)	(−2.60)	(−2.68)	(−1.43)
Overpriced	−0.93	−0.77	−1.40	−1.61	−1.50	−1.82	−2.16	−2.07	−1.89	−2.41	−1.49
	(−1.92)	(−2.01)	(−2.61)	(−3.29)	(−3.49)	(−3.31)	(−4.89)	(−4.30)	(−3.51)	(−4.79)	(−2.04)
All stocks	0.34	−0.01	0.24	0.18	−0.23	0.10	−0.43	−0.19	−0.39	−0.65	−0.99
	(1.59)	(−0.04)	(1.23)	(0.83)	(−1.12)	(0.50)	(−2.24)	(−0.73)	(−1.48)	(−2.09)	(−2.36)

where $R_{i,t}$ and $R_{m,t}$ are the returns on the asset and the market, and $\hat{\beta}_{i,t}$ is the beta estimate (computed using data prior to period t). Assume the asset's return is generated as

$$R_{i,t} = a_i + \beta_{i,t}R_{m,t} + \epsilon_{i,t}, \tag{A.2}$$

where $\mathrm{E}\{\epsilon_{i,t}|R_{m,t}\} = 0$. Define the estimation error in $\hat{\beta}_{i,t}$ as

$$\delta_{it} = \beta_{i,t} - \hat{\beta}_{i,t}, \tag{A.3}$$

and also assume that neither the covariance of $R_{m,t}$ with $\epsilon_{i,t}$ nor the means and variances of $R_{m,t}$ and $\epsilon_{i,t}$ depend on δ_{it}:

$$\begin{aligned} Cov\{R_{m,t}, \epsilon_{i,t}|\delta_{it}\} &= Cov\{R_{m,t}, \epsilon_{i,t}\} = 0 \\ \mathrm{E}\{R_{m,t}|\delta_{it}\} &= \mathrm{E}\{R_{m,t}\} \\ \mathrm{E}\{\epsilon_{i,t}|\delta_{it}\} &= \mathrm{E}\{\epsilon_{i,t}\} = 0 \\ Var\{R_{m,t}|\delta_{it}\} &= Var\{R_{m,t}\} \\ Var\{\epsilon_{i,t}|\delta_{it}\} &= Var\{\epsilon_{i,t}\}. \end{aligned} \tag{A.4}$$

Given the above assumptions, the variance of $h_{i,t}$ conditional on δ_{it} is

$$Var\{h_{i,t}|\delta_{it}\} = \delta_{it}^2 Var\{R_{m,t}\} + Var\{\epsilon_{i,t}\}, \tag{A.5}$$

and, using variance decomposition,

$$\begin{aligned} Var\{h_{i,t}\} &= \mathrm{E}\{Var\{h_{i,t}|\delta_{it}\}\} + Var\{\mathrm{E}\{h_{i,t}|\delta_{it}\}\} \\ &= \mathrm{E}\{\delta_{it}^2\}[Var\{R_{m,t}\} + (\mathrm{E}\{R_{m,t}\})^2] + Var\{\epsilon_{i,t}\} \\ &\quad - (\mathrm{E}\{\delta_{it}\})^2(\mathrm{E}\{R_{m,t}\})^2. \end{aligned} \tag{A.6}$$

Dividing both sides of (A.6) by $Var\{R_{m,t}\}$ gives

$$\begin{aligned} \frac{Var\{h_{i,t}\}}{Var\{R_{m,t}\}} &= \mathrm{E}\{\delta_{it}^2\}[1 + g_t] + \frac{Var\{\epsilon_{i,t}\}}{Var\{R_{m,t}\}} - (\mathrm{E}\{\delta_{it}\})^2 g_t \\ &\approx \mathrm{E}\{\delta_{it}^2\} + \frac{Var\{\epsilon_{i,t}\}}{Var\{R_{m,t}\}}, \end{aligned} \tag{A.7}$$

where

$$g_t = \frac{(\mathrm{E}\{R_{m,t}\})^2}{Var\{R_{m,t}\}}, \tag{A.8}$$

and the approximation in (A.7) invokes the fact that g_t is small with monthly returns. For example, if the monthly market return has mean 0.10/12 and variance $0.20^2/12$, then $g_t = 0.02$.

For any given asset, the second term on the RHS of Eq. (A.7) is equal across different beta-estimation methods because ϵ_{it}, which is the unobserved true disturbance term in (A.2), does not involve beta estimation. Therefore, ranking by the mean squared estimation error in beta, $\mathrm{E}\{\delta_{it}^2\}$, is equivalent to ranking by the variance ratio, $\frac{Var\{h_{i,t}\}}{Var\{R_{m,t}\}}$. Table A.1 reports this ratio, computed over our 1963–2013 sample period, for five sets of value-weighted beta-ranked portfolios, one set for each of the five beta-estimation methods.[6] We construct a set of portfolios for each method because the second RHS term in (A.7) does differ across assets (because ϵ_{it} does). Rather than examine hedging errors on portfolios formed by sorting on betas estimated using just one method, we use all five methods and compute $\frac{Var\{h_{i,t}\}}{Var\{R_{m,t}\}}$ for each of the five sets of beta-sorted decile portfolios. All of these values are reported in Table A.1, and the last row in each panel reports the av-

[6] The variances in the numerator and denominator can change over time, so we estimate those variances using rolling five-year windows and compute the average ratio over our sample period. We value-weight the hedging errors within a decile because our investigation of the beta anomaly examines alphas on value-weighted portfolios. Even though a given error in estimating beta makes a small stock just as likely as a large stock to be put in the wrong beta-sorted portfolio, the small stock gets less weight in our alpha calculations, so the stocks for which accurate beta estimation is more important are those receiving more weight when computing alphas on the beta-sorted portfolios.

erages of $\frac{Var\{h_{i,t}\}}{Var\{R_{m,t}\}}$ across the five sets of decile portfolios. As noted earlier, our method achieves the lowest average of these values for the highest and lowest beta deciles (the bottom right-hand value in each panel of Table A.1).

A.2. Estimating alpha in a conditional-beta setting

Table A.2 repeats the analysis in Table 2, for the same portfolios analyzed there, except that the alphas are estimated using the procedure in Cederburg and O'Doherty (2016).

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